Dictionary Learning Applications in Control Theory

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I. Dictionary Learning

Dictionary learning (DL) for sparse representations (SR) [1] has been one of the central signal processing research topics during the last decade due to its promising results and wide range of applications such as recovery of missing data, face detection, inpainting, classification, and image denoising. In this paper we tailor DL techniques to large NP-hard optimization problems resulted in control applications.

The focal problem is building the sparse signal $\boldsymbol{x}$ by using only a few columns (or atoms) from an overcomplete base $\boldsymbol{D}$ (also called frame or dictionary) to explain the dense signal $\boldsymbol{y}$. With fixed $\boldsymbol{D}$ and known $\boldsymbol{y}$, the search for the best set of atoms can be resumed to the following optimization problem

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \quad & \boldsymbol{y} = \boldsymbol{D}\boldsymbol{x} \\ \text{subject to} \quad & \|\boldsymbol{x}\|_0 \leq s, \end{aligned} \tag{1}$$

where $\|.\|_0$ is the ℓ_0 pseudo-norm counting the non-zero elements of $\boldsymbol{x}$ and s is the sparsity constraint. The algorithm most often used in the literature is OMP, which iteratively chooses one atom at a time such that it is orthogonal to and correlates the most with the existing residual of the current solution. Given a large corpus of signals $\boldsymbol{Y} \in \mathbb{R}^{p \times m}$, the DL process provides us with a specialized representation base $\boldsymbol{D} \in \mathbb{R}^{p \times n}$ which minimizes the sparse approximation error

$$\begin{aligned} \underset{\boldsymbol{D},\boldsymbol{X}}{\text{minimize}} \quad & \|\boldsymbol{Y} - \boldsymbol{D}\boldsymbol{X}\|_F^2 \\ \text{subject to} \quad & \|\boldsymbol{x}_i\|_0 \leq s, \ i = 1:m \\ & \|\boldsymbol{d}_j\|_2 = 1, \ j = 1:n, \end{aligned} \tag{2}$$

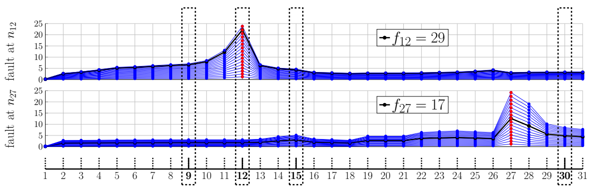
where $\|.\|_F$ is the Frobenius norm and $\boldsymbol{X} \in \mathbb{R}^{n \times m}$ the sparse representation matrix. K-SVD is the de-facto standard for solving (2). Its iterative solution simultaneously updates the current atom and the representations in which it participates through SVD. The algorithm has been adapted for many applications out of which we mention the effective LC-KSVD variation that penalizes (2) in order to obtain class discriminative atoms that are then used for classification.

II. Applications in Control

A. Fault detection and isolation in water networks

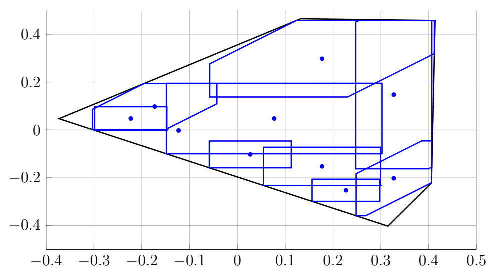
Abnormal behavior (pipe leakage) in distributed water networks is a topic of great interest [2]. The large size of a typical network, its inherent nonlinearity and the limited number of available sensors are significant issues. Thus, sensor placement (SP) and the detection and isolation (FDI) of leakages are difficult problems which require heuristics.

The EPANET emulation benchmark allows to obtain data characterizing the network under various modes of functioning (healthy and faulty with various leakage magnitudes). For SP, selecting atoms from a fixed dictionary allows to place the sensors in the water network. For FDI, the faults affecting a given node represent a class. Training the dictionary such that its atoms discriminate between these classes means that the active atoms for a certain class (fault) can be interpreted as a fault signature. Thus, the fault detect and its subsequent isolation can be interpreted as having a unique combination of active atoms.



B. Area coverage

Keeping the structure of the OMP algorithm (without some of its theoretical properties) it is possible to provide a covering of a given region. Positions from within a given grid are selected and scaling factors for pre-defined zonotopic shapes with common generators are computed, such that the uncovered area is minimized. The result is exploited for the characterization of robust invariant regions associated with switched dynamics [3].



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References

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[3] F. Blanchini and S. Miani, *Set-theoretic methods in control*. Birkhauser, 2007.